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Towards the design of tunable X-ray lasers by dressing the plasma with the elliptically polarized radiation of an optical laser

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Abstract. We present detailed results of calculating the modifications in X-ray lasing lines of a H-like ion under a strong elliptically-polarized electric field of an optical laser. We show that using an elliptically-polarized electric field of an optical laser, it is possible to control the amplification and polarization of the X-ray radiation, and to vary the frequency of the X-ray lasing line.

PACS. 42.55.Vc X- and gamma-ray lasers – 32.70.Jz Line shapes, widths, and shifts – 52.70.La X-ray and gamma-ray measurements

1 Introduction

One of the popular schemes for soft X-ray lasing is the scheme based on recombination pumping of hydrogen-like (H-like) ions which are first stripped of electrons by optical field ionization [1–3]. The gain of an X-ray laser is determined by the product of the width of the lasing line and oscillator strength for the lasing transition. In our earlier works [4,5] we showed that by applying a high frequency electric field of electromagnetic radiation to a H-like emitter, it is possible to substantially decrease the Stark width of its spectral lines. The reason for such a decrease is that the high frequency field changes the character of interaction of a H-like emitter with plasma electric microfield. Based on this idea, it was shown in reference [6] that a linearly-polarized field of an optical laser can significantly narrow the profile of the absorption coefficient for some X-ray lasing spectral lines of H-like ions, thus increasing the gain of the X-ray laser. Later the effect of the narrowing of the line profile for H-like ions interacting with linearly-polarized electric field of an optical laser was also studied in reference [7].

This paper presents the detailed calculations of the effect of the elliptically-polarized intense field of an optical laser (EPIFOL) on the Stark broadening of some X-ray spectral lines of H-like ions. For the interaction of the EPIFOL with a H-like ion, the results obtained in our paper [8] are used. Interaction of a H-like ion with plasma microfields is considered by using the Model Microfield Method [9]. It is shown that the use of the EPIFOL can provide not only a significant enhancement of the gain on some X-ray lasing transitions, but also a possibility of tuning the X-ray laser in a wide range of frequencies.

2 Theoretical model

We consider a H-like ion of a nuclear charge Z subjected simultaneously to an elliptically-polarized electric field of an optical laser

$$\vec{E}(t) = \varepsilon_0[\vec{e}_z \cos(\omega_0 t + \gamma) + \xi\, \vec{e}_x \sin(\omega_0 t + \gamma)] \qquad (1)$$

and to the plasma microfield $\vec{F}(t)$. In equation (1), $\vec{e}_x$ and $\vec{e}_z$ are unit vectors along x- and z-axes of a Cartesian reference frame, ξ is the ellipticity degree. For a H-like ion under the field $\vec{E}(t)$, the Wave Functions (WFs) of quasienergy states for the energy level of the principal quantum number $n = 2$ were found in [8] in the following form:

$$\begin{aligned}
&\Psi_1 \equiv \tilde{\psi}_1(t),\ \Psi_2 \equiv \tilde{\psi}_2(t),\ \Psi_3 \equiv \exp(-i\kappa t)\tilde{\psi}_3(t),\\
&\Psi_4 \equiv \exp(i\kappa t)\tilde{\psi}_4(t),\\
&\tilde{\psi}_k(t) = 2^{-1}\{(-1)^{k+1}(\varphi_1 - \varphi_2) + i\varphi_3 \exp[-i\beta(t)]\\
&\qquad + i\varphi_4 \exp[i\beta(t)]\},\\
&\tilde{\psi}_p(t) = 2^{-1}\{i(\varphi_1 + \varphi_2) + (-1)^{p+1}\{\varphi_3 \exp[-i\beta(t)]\\
&\qquad - \varphi_4 \exp[i\beta(t)]\}\},\\
&k = 1, 2; \quad p = 3, 4,
\end{aligned} \qquad (2)$$

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where

$$\kappa = \omega_0 \xi w J_1(w), \quad \beta = w \sin(\omega_0 t + \gamma), \quad w = 3\varepsilon_0/(Z\omega_0). \tag{3}$$

Here $J_1(w)$ is the Bessel function and the functions φ_s ($s = 1, 2, 3, 4$) are the parabolic WFs of the level $n = 2$ of the H-like ion ($\varphi_1 \equiv |001\rangle$, $\varphi_2 \equiv |00-1\rangle$, $\varphi_3 \equiv |100\rangle$, $\varphi_4 \equiv |010\rangle$), O$z$ being the quantization axis. The WFs (2) were obtained in [8] for the case of $|\xi| \le w^{-1/2}$. In equation (3) and below we use atomic units $\hbar = m_e = e = 1$, unless specified to the contrary.

The Schrödinger equation for the H-like ion under the field $\vec{E}(t) + \vec{F}(t)$ can be written as

$$i\frac{\partial \Phi}{\partial t} = [H_a + z\varepsilon_0 \cos(\omega_0 t + \gamma) + x\varepsilon_0 \xi \sin(\omega_0 t + \gamma) + V(t)]\Phi,$$
$$V(t) \equiv \vec{r} \cdot \vec{F}(t). \tag{4}$$

Here H_a is the Hamiltonian of the unperturbed ion.

We seek a solution of equation (4) for $n = 2$ in the form

$$\Phi_j(t) = \sum_r T_{rj}(t,0)\tilde{\psi}_r(t) \tag{5}$$

for initial conditions

$$T_{rj}(0,0) = \delta_{rj}$$

where δ_{rj} is the Kronecker symbol. The matrix $T_{rj}(t,0)$ consists of matrix elements of the evolution operator $T(t,0)$, $T(0,0)$ being the unity operator I.

Substituting (5) in (4) we obtain

$$\frac{dT}{dt} = M(\vec{F}, t)T \tag{6}$$

where elements of the matrix $M(\vec{F}, t)$ are

$$M_{qr}(\vec{F}, t) = -i\langle \tilde{\psi}_q(t) \, |V(t)| \, \tilde{\psi}_r(t)\rangle. \tag{7}$$

In this paper we use the Model Microfield Method (MMM) for describing the interaction of the plasma microfield with H-like ions [9]. For using the MMM, it is important to find a static evolution operator T_S, i.e., the evolution operator for the case of a time-independent field $\vec{F}$. Here we consider the situation where the optical laser frequency ω_0 is much greater than the Stark splitting of the of the level $n = 2$ of the H-like ion in the field F

$$\omega_0 \gg 3F/Z. \tag{8}$$

Under the condition (8), we use the averaging method by Krylov-Bogoliubov-Mitropolskii [10,11] for solving the matrix equation (6) for the time-independent field $\vec{F}$. In accordance to this averaging method, an approximate solution of equation (6) is obtained via the substitution of the matrix elements $M_{qr}(\vec{F}, t)$ by their time-averaged values $\overline{M}_{qr}(\vec{F})$:

$$M_{qr}(\vec{F}, t) \to \overline{M}_{qr}(\vec{F}),$$
$$\overline{M}_{qr}(\vec{F}) = \frac{\omega_0}{2\pi} \int_0^{2\pi/\omega_0} M_{qr}(\vec{F}, t)dt. \tag{9}$$

As a result we get the following equation for finding the static evolution operator $T_S(t)$

$$\frac{dT_S}{dt} = \overline{M}(\vec{F})T_S, \tag{10}$$

the matrix $\overline{M}(\vec{F})$ for the level $n = 2$ of the H-like ion being

$$\overline{M}(\vec{F}) = \begin{pmatrix} if_y & 0 & if_x - f_z & if_x + f_z \\ 0 & -if_y & if_x - f_z & if_x + f_z \\ if_x + f_z & if_x + f_z & -i\kappa & 0 \\ if_x - f_z & if_x - f_z & 0 & i\kappa \end{pmatrix} \tag{11}$$

where

$$f_x = \frac{3}{2Z}J_0(w)F_x, \quad f_y = \frac{3}{Z}J_0(w)F_y, \quad f_z = \frac{3}{2Z}F_z. \tag{12}$$

Here $J_0(w)$ is the Bessel function. The principal characteristics of the model microfield $\vec{F}$, which controls the line profile in the MMM, are the distribution function $P(\vec{F})$ and the jumping frequency $\nu(\vec{F})$ [9]. Between two consecutive jumps, the field $\vec{F}$ is considered to be time-independent.

The MMM equation for the Fourier transform of the transition operator averaged over the realizations of the stochastic plasma microfield $\vec{F}(t)$ is as follows [9]

$$\tilde{T}_{MMM}(\omega) = \{\tilde{T}_S(\tilde{\omega})\}_{\rm av} + \{\nu\tilde{T}_S(\tilde{\omega})\}_{\rm av}\{\nu I - \nu^2\tilde{T}_S(\tilde{\omega})\}_{\rm av}^{-1}\{\nu\tilde{T}_S(\tilde{\omega})\}_{\rm av}. \tag{13}$$

Here $\nu = \nu(\vec{F})$; $\tilde{T}_S(\tilde{\omega})$ is the Laplace transform of the transition operator $T_S(t)$ calculated at $\tilde{\omega} = \omega + i\nu(\vec{F})$ for a static field $\vec{F}$; $\{...\}_{\rm av}$ is the average over the probability distribution $P(\vec{F})$.

Using equation (13), we can represent the spectral line profile of the L_α line of the H-like ion in the form

$$I^{(\zeta)}(\omega) = \frac{1}{\pi}\sum_{\alpha',\alpha''} {\rm Re}[\tilde{T}_{MMM}(\omega)]_{\alpha'',\alpha'} R^{(\zeta)}_{\alpha'\alpha''} \tag{14}$$

where ζ indicates the polarization of the emitted photons ($\zeta = x, y, z$), and

$$R^{(\zeta)}_{\alpha'\alpha''} = \left\{ \left\langle \varphi_0 \, |\zeta| \, \tilde{\psi}^{(0)}_{\alpha''}(\tau) \right\rangle \left\langle \tilde{\psi}^{(0)}_{\alpha'}(0) \, |\zeta| \, \varphi_0 \right\rangle \right\}_\gamma. \tag{15}$$

In equation (15), $\{...\}_\gamma$ is the average over the initial phase γ of the laser field $\vec{E}(t)$, $\tilde{\psi}^{(0)}_\alpha(\tau)$ is the zero harmonic of the periodic WF $\tilde{\psi}_\alpha(\tau)$, and φ_0 is the WF of the lower level $n = 1$.

3 Calculation of the modified profile of the absorption coefficient of the lasing line L_α

In this paper we present calculations for the L_α line of Li III ($\lambda = 135$ Å) subjected to an elliptically-polarized

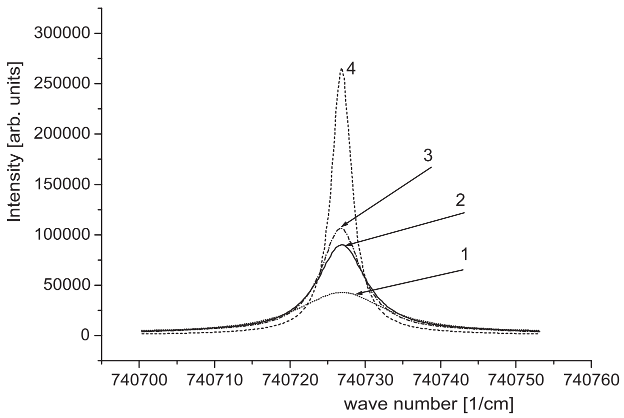

Fig. 1. Modification of the profile of the Li III L_α line ($\lambda =$ 135 Å) under a *linearly*-polarized electric field $\vec{E}(t)$ of the CO_2 laser in a plasma of the electron density $N_e = 5.0 \times 10^{18}$ cm^{-3} and of the temperature $T_e = T_i = 3$ eV. Perturbing ions are Li^{3+}. Profiles 1, 2, and 4 correspond to the dressing field amplitude 0, 3.36×10^7 V/cm, and 4.51×10^7 V/cm, respectively. The X-ray radiation is polarized perpendicularly to direction of the CO_2 laser field. Profile 3 was calculated at the dressing field amplitude 3.36×10^7 V/cm (like profile 2), but allowing only for the broadening by ions — to illustrate the fact that, under the considered plasma parameters, the ionic contribution to the width of the profile of the absorption coefficient predominates over the electronic contribution.

electric field $\vec{E}(t)$ of the CO_2 laser in a plasma of the electron density $N_e = 5.0 \times 10^{18}$ cm^{-3} and at the temperature $T_e = T_i = 3$ eV. We note that the chosen plasma parameters are close to the experimental conditions of reference [2]. For calculating the distribution function $P(\vec{F})$ we employed analytical formulas from reference [12]. These formulas are based on using the results of calculating $P(\vec{F})$ by the adjustable-parameter exponential approximation (APEX) [13,14]. For the calculation of the jumping frequency $\nu(\vec{F})$ we utilized results from reference [15].

Figure 1 shows the modification of the Li III L_α line profile under a *linearly*-polarized field $\vec{E}(t)$ (i.e., for $\xi = 0$) for the following three values of the laser field amplitude ε_0: 0 (profile #1); 3.36×10^7 V/cm (profile #2), and 4.51×10^7 V/cm (profile #4). We assumed that the polarization of the L_α radiation is perpendicular to the vector $\vec{E}(t)$. It is seen that as the value ε_0 increases within the above range, the profile of the absorption coefficient of the L_α line becomes narrower — in accordance to the earlier results [4–6].

The ratio of halfwidths (FWHM) $\Delta\omega_{1/2}^{(k)}$ for these three profiles in Figure 1 is:

$$\Delta\omega_{1/2}^{(4)} : \Delta\omega_{1/2}^{(2)} : \Delta\omega_{1/2}^{(1)} = 0.21 : 0.54 : 1.00,$$

where $\Delta\omega_{1/2}^{(1)}$, $\Delta\omega_{1/2}^{(2)}$, and $\Delta\omega_{1/2}^{(4)}$ correspond to $\varepsilon_0 = 0$, $\varepsilon_0 = 3.36 \times 10^7$ V/cm, and $\varepsilon_0 = 4.51 \times 10^7$ V/cm, respectively.

Due to the narrowing of the profile of the absorption coefficient of the L_α line under the increase of ε_0, the intensity at the maximum of the normalized profile increases. For a lasing plasma this leads to the growth of the amplification and of the gain of the X-ray laser under the dressing by a *linearly*-polarized radiation of an optical laser. This phenomenon was studied earlier in [6].

Figure 1 also shows the profile #3, calculated at $\varepsilon_0 = 3.36 \times 10^7$ V/cm (like the profile #2), but allowing only for the broadening by ions. The comparison of the profiles #2 and #3 illustrates the fact that, under the considered plasma parameters, the ionic contribution to the width of the profile of the absorption coefficient predominates over the electronic contribution. Our calculations showed that, for the L_α line of Li III under the considered plasma parameters, the Stark broadening by ion microfields is about two times greater than the Stark broadening by electron microfields.

Figure 2 shows the modification of the Li III L_α line profiles of x-, y-, and z-polarizations subjected to the CO_2 laser of $\varepsilon_0 = 3.36 \times 10^7$ V/cm as the *ellipticity* degree increases from $\xi = 0$ to $\xi = 0.2$. Since at the chosen plasma parameters, the pre-dominant effect in the Stark broadening of the Li III L_α line is due to the ion microfield, we did not allow for the electron microfield while calculating profiles presented in Figure 2. The most interesting is the profile of the x-polarization, i.e., the profile polarized in the direction of the minor component of the field $\vec{E}(t)$. This profile shows a splitting into three components: the central component at the unperturbed frequency $\omega_{21}^{(0)}$ of the L_α transition and two lateral components at the frequencies $\omega_{21}^{(0)} \pm \kappa$, where κ is defined in equation (3). For the case under the consideration, at $\xi \geq 0.05$ the lateral components become more intensive in their maxima than the central component. Therefore we can expect that under certain condition it is possible to generate the X-ray radiation of the x-polarization at the shifted frequencies $\omega_{21}^{(0)} \pm \kappa$. Since the quasienergy κ is proportional to the ellipticity degree ξ (see Eq. (3)), then one can design a *tunable* X-ray laser, where the tuning would be achieved by varying ξ.

Another interesting result is that via the dressing by an elliptically-polarized field of an optical laser, it is possible to *control the polarization* of the generated X-ray radiation. Indeed, the employment of the elliptically-polarized dressing field produces the X-ray radiation (at the shifted frequencies $\omega_{21}^{(0)} \pm \kappa$) polarized along the minor component of the field $\vec{E}(t)$ (i.e., along the minor axis of the ellipse). As for achieving the maximum generation at the unshifted frequency $\omega_{21}^{(0)}$, one should use instead a linearly-polarized dressing field $\vec{E}(t)$. In the latter case, the X-ray radiation would be polarized perpendicular to the dressing field $\vec{E}(t)$.

4 Discussion

In this paper we studied how the X-ray lasing at the L_α line of an H-like ion can be controlled via a dressing by an elliptically-polarized radiation of an optical laser.

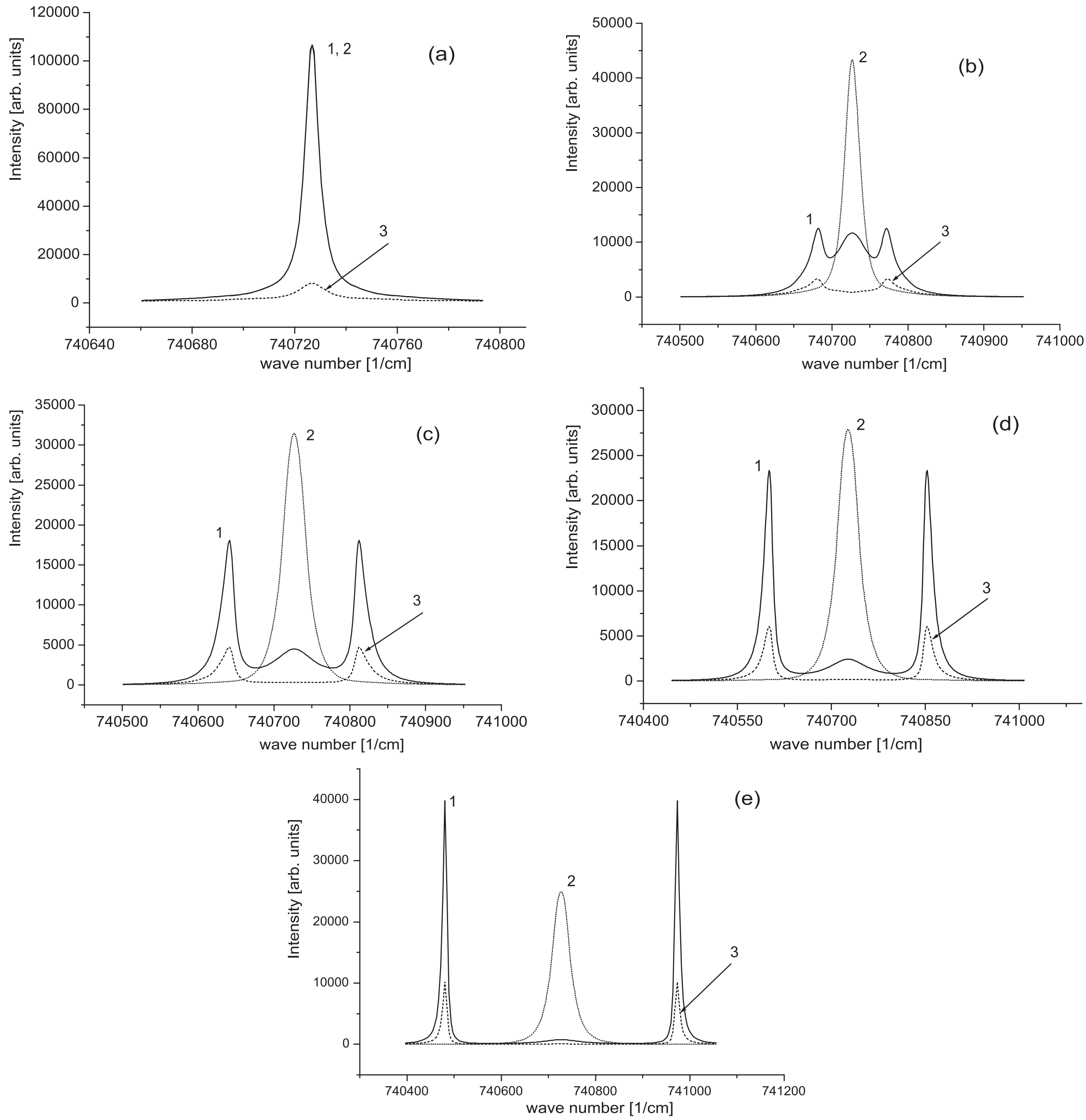

Fig. 2. Modification of the profile of the Li III L_α line ($\lambda = 135$ Å) under an *elliptically*-polarized electric field $\vec{E}(t)$ of the CO_2 laser in a plasma of the electron density $N_e = 5.0 \times 10^{18}$ cm^{-3} and of the temperature $T_e = T_i = 3$ eV. The dressing field amplitude is 3.36×10^7 V/cm. Perturbing ions are Li^{3+}. The sub-figures (a, b, c, d, e) correspond to the ellipticity degree 0, 0.05, 0.10, 0.15, and 0.20, respectively. In each of the sub-figures, profiles 1, 2, and 3 correspond to the X-ray radiation polarized in x-, y-, and z-directions, respectively, where z is the direction of the major axis of the ellipse and x is the direction of the minor axis of the ellipse (see Eq. (1)).

We showed first of all that such a dressing can *control the amplification* of the X-ray radiation at the unperturbed frequency $\omega_{21}^{(0)}$ of the L_α line. In particular, the maximum amplification can be achieved for the ellipticity degree $\xi = 0$ and it would be significantly greater than at the absence of the dressing field. Second, by using an elliptically-polarized radiation of an optical laser of $\xi > 0$, it is possible *to design a tunable X-ray laser*: the tuning would be performed by varying the ellipticity degree ξ. Third, such a dressing allows to *control the polarization* of the X-ray radiation.

We performed detailed calculations for the radiating H-like Li ion assuming that perturbing ions are Li^{3+}.

Let us analyze the role of the nuclear charge Z of the radiating ion. The controlling parameter in the above effects is the dimensionless quantity $w = 3\varepsilon_0/(Z\omega_0)$ defined in equation (3). First, this parameter controls positions of the lateral spectral components given by the quantity $\kappa = \omega_0 \xi w J_1(w)$ (also defined in Eq. (3)). Second, the parameter w enters the effective static electric field $f_x = (3/2Z)J_0(w)F_x$, $f_y = (3/Z)J_0(w)F_y$ (defined by Eq. (12)), so that it controls the Stark broadening of spectral components.

The Bessel functions $J_0(w)$ and $J_1(w)$ are oscillatory functions of their argument. Therefore, at fixed parameters of the dressing optical laser (so that the ratio ε_0/ω_0 is fixed), for $w > 1$ both the positions of the lateral components and their Stark broadening vary non-monotonically with the nuclear charge Z. For $w < 1$, as Z increases, positions of the lateral components come closer to the unperturbed position of the Lyman-alpha line (the quantity $\kappa = \omega_0 \xi w J_1(w)$ decreases); the Stark broadening of the spectral components diminishes as well.

The validity condition for the obtained results is given by equation (8): $\omega_0 \gg 3F/Z$. This means that the frequency of the dressing optical laser should be sufficiently high. For this condition to be met for the majority of radiating H-like ions in a plasma, the electron density N_e should not be too high. Substituting the characteristic ion microfield $F_0 = 2.6eZ^{1/3}N_e^{2/3}$ in equation (8), we can rewrite it in the form (in CGS units):

$$\omega_0 \gg 8\hbar Z^{-2/3} N_e^{2/3}/m_e. \tag{16}$$

For the plasma parameters $N_e = 5.0 \times 10^{18}$ cm^{-3} and $Z = 3$, that we used as an example (because they are close to the experimental conditions of Ref. [2]), equation (16) translates into $\omega_0 \gg 1.3 \times 10^{13}$ s^{-1}. This requirement is satisfied with a large margin for a CO_2 laser employed as a dressing radiation source.

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